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ON THE SOLUTION OF KADOMTSEV –PETVIASHVILI EQUATION USING DIRECT SUBSTITUTION METHOD

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Abstract
We study Solutions of Kadomtsev – Petviashvili Equation with boundary condition on Cartesian Co-ordinates, we also study Seven Parameter lie group. In this paper, we discussed Kadomtsev – Petviashvili Equation in xydu and using partial differential equation.

1.Introduction

Symmetry analysis in nowdays developed to find solution of symmetry differential equation . after symmetry of a differential equation the solution transformed and maps new system of solution . it is very helpful to many field . In this paper, we have to find seven parameter and also commutator table of Kadomtsev–Petviashvili Equation equation. It is known that all physical phenomena can be described through nonlinear partial differential equations, so finding exact solutions to these equations and studying them represents the cornerstone through which we can better understand the mechanisms of the complex physical phenomena that these equations represent. They enable us also to clearly understand the dynamic processes these equations accompaniment. Moreover, those exact solutions can also help to investigate the stability of these solutions and to test numerical analysis for these nonlinear partial differential equations. In recent years, the reduction of partial differential equations (PDEs) into ordinary differential equations (ODEs) has proven a successful idea for constructing interesting exact solutions of the nonlinear differential equations. The Lie Group method is a basic and powerful tool for
obtaining symmetries for differential equations, which used for reducing the differential equations and obtaining the exact solutions [1–6]. The mathematical method in the present study is the one-parameter group transformation. In the Lie group method, the infinitesimal functions which consist of independent and dependent variables have been presented for expressing the infinitesimals. Finding the infinitesimal functions is the first and basic step in the Lie group method, through these functions the auxiliary equation can be constructed and thus we get the invariant symmetries. The infinitesimal functions are calculated by solving a system of linear partial differential equations called the determining equations that arise by applying the invariance conditions to the partial differential equations and their auxiliary conditions. Therefore, the Lie group method can be used easily to solve different types of nonlinear problems. The major characteristic of Lie group method is decreasing the number of independent variables for the partial differential equations by one. Thus, the obtained symmetries were used to reduce the negative-order Kadomtsev– Petviashvili equation to an ordinary differential equation [7]

Symmetries of Kadomtsev – Petviashvili Equation

Consider the Kadomtsev – Petviashvili equation

$$u_{tx} + 3u_{yy} + 3(u^2)_{xx} + u_{xxxx} = 0 \qquad \to (1.1)$$

In expanded form KP equation in (5.1) reads,

$$u_{tx} + 3u_{yy} + 6uu_{xx} + 6u_x^2 + u_{xxxx} = 0 \qquad \to 1.2)$$

The infinitesimal generator of a point symmetry is given by,

$$X1 = \xi_1(x,y,t,u)\frac{\partial}{\partial x} + \xi_2(x,y,t,u)\frac{\partial}{\partial y} + \tau(x,y,t,u)\frac{\partial}{\partial t} + \eta(x,y,t,u)\frac{\partial}{\partial u} \quad \rightarrow (1.3)$$

For PDE (1.1) the symmetry determining equation is

$$\eta_{tx}^{(2)} + 3\eta_{yy}^{(2)} + 6u\eta_{xx}^{(2)} + 6u_{xx}\eta + 12u_x\,\eta_x^{(1)} + \eta_{xxxx}^{(4)} = 0 \quad \rightarrow (1.4)$$

$$\eta_x^{(1)} = \frac{\partial\eta}{\partial x} - \frac{\partial\xi_1}{\partial x}u_x - \frac{\partial\xi_2}{\partial x}u_y - \frac{\partial\tau}{\partial x}u_t \quad \rightarrow (1.5)$$

$$\eta_y^{(1)} = \frac{\partial\eta}{\partial y} - \frac{\partial\xi_1}{\partial y}u_x - \frac{\partial\xi_2}{\partial y}u_y - \frac{\partial\tau}{\partial y}u_t \quad \rightarrow (1.6)$$

$$\eta_t^{(1)} = \frac{\partial\eta}{\partial t} - \frac{\partial\xi_1}{\partial t}u_x - \frac{\partial\xi_2}{\partial t}u_y - \frac{\partial\tau}{\partial t}u_t \quad \rightarrow (1.7)$$

$$\eta_{xx}^{(2)} = \frac{\partial^2\eta}{\partial x^2} - \frac{\partial^2\xi_1}{\partial x^2}u_x - \frac{\partial^2\xi_2}{\partial x^2}u_y - \frac{\partial^2\tau}{\partial x^2}u_t - 2\frac{\partial\xi_1}{\partial x}u_{xx} - 2\frac{\partial\xi_2}{\partial x}u_{xy} - 2\frac{\partial\tau}{\partial x}u_{xt} \quad \rightarrow (1.8)$$

$$\eta_{yy}^{(2)} = \frac{\partial^2\eta}{\partial y^2} - \frac{\partial^2\xi_1}{\partial y^2}u_x - \frac{\partial^2\xi_2}{\partial y^2}u_y - \frac{\partial^2\tau}{\partial y^2}u_t - 2\frac{\partial\xi_1}{\partial y}u_{yx} - 2\frac{\partial\xi_2}{\partial y}u_{yy} - 2\frac{\partial\tau}{\partial y}u_{yt} \quad (1.9)$$

$$\eta_{tx}^{(2)} = \frac{\partial^2\eta}{\partial t\partial x} - \frac{\partial^2\xi_1}{\partial t\partial x}u_x - \frac{\partial^2\xi_2}{\partial t\partial x}u_y - \frac{\partial^2\tau}{\partial t\partial x}u_t - \frac{\partial\xi_1}{\partial x}u_{xx} - \frac{\partial\xi_1}{\partial x}u_{tx} - \frac{\partial\xi_2}{\partial t}u_{xy}$$
$$- \frac{\partial\xi_2}{\partial x}u_{ty} - \frac{\partial\tau}{\partial x}u_{tt} - \frac{\partial\tau}{\partial t}u_{xt} \quad \rightarrow (1.10)$$

$$\eta_{xxx}^{(3)} = \frac{\partial^3\eta}{\partial x^3} - \frac{\partial^3\xi_1}{\partial x^3}u_x - 3\frac{\partial^2\xi_1}{\partial x^2}u_{xx} - \frac{\partial^3\xi_2}{\partial x^3}u_y$$
$$-3\frac{\partial^2\xi_2}{\partial x^2}u_{xy} - \frac{\partial^3\tau}{\partial x^3}u_t - 3\frac{\partial^2\tau}{\partial x^2}u_{xt}$$
$$-3\frac{\partial\xi_1}{\partial x}u_{xxx} - 3\frac{\partial\xi_2}{\partial x}u_{xxy} - 3\frac{\partial\tau}{\partial x}u_{xxt} \quad \rightarrow (1.11)$$

$$\eta_{xxxx}^{(4)} = \frac{\partial^4\eta}{\partial x^4} - \frac{\partial^4\xi_1}{\partial x^4}u_x - \frac{\partial^4\xi_2}{\partial x^4}u_y - \frac{\partial^4\tau}{\partial x^4}u_t$$
$$-4\frac{\partial^3\xi_1}{\partial x^3}u_{xx} - 4\frac{\partial^3\tau}{\partial x^3}u_{xt} - 4\frac{\partial^3\xi_2}{\partial x^3}u_{xy}$$
$$-6\frac{\partial^2\xi_1}{\partial x^2}u_{xxx} - 6\frac{\partial^2\xi_2}{\partial x^2}u_{xxy} - 6\frac{\partial^2\tau}{\partial x^2}u_{xxt}$$
$$-4\frac{\partial\xi_1}{\partial x}u_{xxxx} - 4\frac{\partial\xi_2}{\partial x}u_{xxxy} - 4\frac{\partial\tau}{\partial x}u_{xxxt} \quad \rightarrow (1.12)$$

Substituting (1.10), (1.9), (1.8), (1.5) & (1.12) into (1.3) the system of determining equations for $\{\xi_1, \xi_2, \tau, \eta\}$ are,

$$\frac{\partial\xi_1}{\partial u} = o, \frac{\partial\xi_2}{\partial u} = 0, \frac{\partial\tau}{\partial u^2} = 0, \frac{\partial^2\eta}{\partial u^2} = 0 \quad \rightarrow (1.13)$$

$$\frac{\partial\tau}{\partial x} = 0, \frac{\partial\tau}{\partial y} = 0 \quad \rightarrow (1.14)$$

$$\frac{\partial^2\xi_1}{\partial x^2} = 0 \quad \rightarrow (1.15)$$

$$\frac{\partial^2\eta}{\partial x\partial u} = 0 \quad \rightarrow (1.16)$$

$$\frac{\partial^2\eta}{\partial x\partial t}+2u\frac{\partial^2\eta}{\partial x^2}+3\frac{\partial^2\eta}{\partial y^2}+\frac{\partial^4\eta}{\partial x^4}=0 \qquad \rightarrow (1.17)$$

$$4\frac{\partial\eta}{\partial x}-\frac{\partial\xi_1}{\partial x\partial t}-3\frac{\partial^2\xi_1}{\partial y^2}+\frac{\partial^2\eta}{\partial t\partial u}+4\frac{\partial^4\eta}{\partial x^4}=0 \qquad \rightarrow (1.18)$$

$$2\eta-\frac{\partial\xi_1}{\partial t}-4u\frac{\partial\xi_1}{\partial x}+2u\frac{\partial\eta}{\partial u}+6\frac{\partial^3\eta}{\partial x^3}=0 \qquad \rightarrow (1.19)$$

$$-\frac{\partial\xi_1}{\partial x}-\frac{\partial\xi_3}{\partial t}+\frac{\partial\eta}{\partial u}=0 \qquad \rightarrow (1.20)$$

$$-\frac{\partial\xi_1}{\partial x}+\frac{\partial\eta}{\partial u}=0 \qquad \rightarrow (1.21)$$

$$-4\frac{\partial\xi_1}{\partial x}+\frac{\partial\eta}{\partial u}=0 \qquad \rightarrow (1.22)$$

$$-\frac{\partial^2\xi_2}{\partial y^2}+2\frac{\partial^2\eta}{\partial y^2}=0 \qquad \rightarrow (1.23)$$

$$-2\frac{\partial\xi_2}{\partial y}+\frac{\partial\eta}{\partial u}=0 \qquad \rightarrow (1.24)$$

$$-6\frac{\partial\xi_1}{\partial y}-\frac{\partial\xi_2}{\partial t}=0 \qquad \rightarrow (1.25)$$

The solutions of the determining equations are given by,

$$\begin{aligned}
\xi_1 &= 2t\alpha_1+\alpha_2-\alpha_3 y+(6tx-y^2)\,\alpha_5+\alpha_6 x\\
\xi_2 &= 6\alpha_3 t+\alpha_4+12\alpha_5\,ty+2\alpha_6 y \qquad \rightarrow (1.26)\\
\tau &= 9\alpha_5 t^2+3\alpha_6 t+\alpha_7\\
\eta &= (-12\alpha_5 t-2\alpha_6)\,u+3\alpha_5 x+\alpha_1
\end{aligned}$$

Hence the point symmetry generators admitted by (1.1) are given by,

$$\begin{aligned}
X_1 &= 2t\frac{\partial}{\partial x}+\frac{\partial}{\partial u}\\
X_2 &= \frac{\partial}{\partial x}\\
X_3 &= -y\frac{\partial}{\partial x}+6t\frac{\partial}{\partial y}\\
X_4 &= \frac{\partial}{\partial y} \qquad \rightarrow (1.27)\\
X_5 &= (6tx-y^2)\frac{\partial}{\partial x}+12ty\frac{\partial}{\partial y}+9t^2\frac{\partial}{\partial t}+(-12tu+3x)\frac{\partial}{\partial u}\\
X_6 &= x\frac{\partial}{\partial x}+2y\frac{\partial}{\partial y}+3t\frac{\partial}{\partial t}-2u\frac{\partial}{\partial u}\\
X_7 &= \frac{\partial}{\partial t}
\end{aligned}$$

The commutator table for the Lie algebra arising from (1.27) is given by,

	X_1	X_2	X_3	X_4	X_5	X_6	X_7
X_1	0	0	0	0	$-3tX_1$	$-2X_1$	$-2X_2$
X_2	0	0	0	0	$3X_1$	X_2	0
X_3	0	0	0	X_2	$-3yX_1 + 18t^2X_4$	$-X_3$	$-6X_4$
X_4	0	0	$-X_2$	0	$2X_3$	$2X_4$	0
X_5	$3tX_1$	$-3X_1$	$3yX_1 - 18t^2X_4$	$-2X_3$	0	$-3X_5$	$-6X_6$
X_6	$2X_1$	$-X_2$	X_3	$-2X_4$	$3X_5$	0	$-3X_7$
X_7	$2X_2$	0	$6X_4$	0	$6X_6$	$3X_7$	0

CONCLUSION

In this Paper , we find the lie symmetries of Kadomtsev – Petviashvili Equation by using introducing variables and operator for finding solution of Kadomtsev – Petviashvili Equation and seven parameter and also satisfy lie symmetry commutator table

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